Supplier Evaluation in a Supply Chain Using Fuzzy Multi Criteria Decision Making

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Abstract -Effectively selecting and evaluating suppliers and managing their involvement in critical supply chain activities play vital roles in building competitive supply chains. This research proposes improved model for optimal supplier selection in case of a multi-suppliers for supply chain. So to formulate decision model for evaluation of suppliers through supply echelon that helps to improve the ability of supply chain in facing aggressive competition with other competitors, and that will be imposed from decision models tradeoff between the total monetary cost and measures for supplier performance service level across supply echelon. So, a Multi-Criteria Decision Making (MCDM) techniques with fuzzy theory for evaluating and obtaining weights for alternatives and criteria was proposed. By using Fuzzy Analytical Hierarchy Process (F-AHP) and using fuzzy preference programming to handle with fuzzy judgments comparison matrices in fuzzy AHP and using Technique for Order Preferences by Similarity to Ideal Solution (TOPSIS) as another technique for results comparison.

Keywords- Analytical Hierarchy Process (AHP), Fuzzy Preference Programming (FPP), Technique for Order Preferences by Similarity to Ideal Solution (TOPSIS), Supplier selection, Multi Criteria Decision Making(MCDM).

1 Introduction

Supply chain management (SCM) is the oversight of materials, information, and finances as they move in a process from supplier to manufacturer to wholesaler to retailer to consumer. Supply chain management involves coordinating and integrating these flows both within and among companies. Today's competitive environment in real supply chain, companies are trying to reach the goals of introducing good competitive and responsive system through low cost, high quality, flexibility and more customer satisfaction. Managing the sourcing issues in the supply chain has been a challenge in the last decade for many corporations [1]. So the successful supply chain management requires an effective and efficient sourcing strategy to eliminate the uncertainties in both supply and demand. The industrial corporations are led to adopt the supply chain management (SCM) philosophy to cope with market challenges [2].

Sourcing decisions are more critical than ever, since with the increase of the purchasing costs in the overall costs, the purchasing function and the purchasing decisions have gained a considerable importance at each firm. On average, a typical manufacturing company spends 60% of its total turnover in purchasing materials, goods and services acquired from external suppliers [3]. Thus purchasing decisions which are related to supplier selection have significant effects on lowering costs and increasing profits.

One of the most important processes of the purchasing functions is the supplier selection [4]. The identification and evaluation of multi suppliers for selecting an appropriate supplier to ensure that a firm will receive high-quality materials at a reasonable price, deliver the right quantities at the right time and provide excellent services in order to satisfy customers' demands [5]. Thus the purchasing department can plays a key role in an organization's efficiency and effectiveness because it has a direct effect on cost reduction, profitability and flexibility of a company [6]. Thus the supplier selection process is one of the most important components of production and logistics management for many companies. Selection of a wrong supplier could be enough to make a negative influence on the company's financial and operational position. Selecting the right suppliers significantly reduces purchasing costs, improves competitiveness in the market and enhances end user satisfaction. The supplier selection process mainly involves evaluation of different alternative suppliers based on different criteria. This process is essentially considered as Multiple Criteria Decision-Making (MCDM) problem which is affected by different tangible and intangible criteria including price, quality, performance, technical capability, delivery, etc. Based on several criteria and alternatives to be considered, various decision making methods have been proposed to provide a solution to this problem [7].

Analytical Hierarchy Process (AHP), which was first developed by Saaty [8], integrates expert's opinions and evaluation scores into a simple elementary hierarchy system by decomposing complicated problems from higher hierarchies to lower ones. Yahya and Kingsman [9] are one of the first known researchers to use AHP to determine priorities in selecting suppliers. Ghodseypour and O'Brien [10] applied the method of integrating AHP and linear programming for the first time to make a tradeoff between tangible and intangible factors with different priorities. After that, Tam and Tummala [11] formulated an AHP-based model and applied it to a telecommunication system.

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As a pioneer in the supplier selection problem, Dickson [12] identified 23 different criteria for selecting suppliers, including quality, delivery, performance history, warranties, price, technical capability and financial position, Jolai [13]. In 2001, Akarte [14] has proposed a systematic methodology to evaluate suppliers using AHP, which has been based on 18 subjective and objective criteria. Also Chan [15] used AHP approach considering the goals of cost, quality, technology, performance and design. Chan [16] showed that the vendor selection problem must be solved in a structural manner and provide a framework for the organization to select suppliers using AHP. Yang and Chen [17] applied an integrated model of AHP and grey relational analysis (GRA) method to a real case to examine its flexibility in selecting the best supplier. Liu and Hai al. [18] used AHP method but instead of pair-wise comparison, they applied the voting method. Hou and Su [19] applied AHP and the business theories to provide a web-based supplier selection system. Also chan et al. [20] presented a case study on solving the supplier selection problem in the airline industry through a decision support system that employs the analytical hierarchy process. Ramantahan [21] integrated AHP and the total cost of ownership approach to consider mix of both qualitative and quantitative factors in supplier selection process.

However since the uncertainty and vagueness of the expert's opinion is the prominent characteristic of the problem, this impreciseness of human's judgments can be handled through the fuzzy sets theory developed by Zadeh [22]. Basically, Fuzzy AHP method represents the elaboration of a standard AHP method into fuzzy domain by using fuzzy numbers for calculating instead of real numbers [23]. Micheli et al. [24] used AHP approach to create a systematic framework to examine the strength and weakness of a vendor's capability using fuzzy values. Kahraman et al. [25] used AHP method in a fuzzy environment and Chan and et [26] presented a model to select the best global supplier using triangular fuzzy numbers to construct fuzzy pair-wise comparison.

In 2010, a Fuzzy AHP method is used for supplier selection in electronic market places [27]. According to the two phase methodology, , initial screening of the suppliers through the enforcement of hard constraints on the selection criteria is performed at the first phase. In the second phase, final supplier evaluation is performed through the application of a modified variant of Fuzzy AHP. This methodology facilitates an easier elicitation of user preferences through the reduction of necessary user input (i.e. pair wise comparisons) and reduces computational complexity.

In 2011, Fuzzy AHP approach is used for supplier selection in a washing machine company [28]. First they determine the criteria providing the most customer satisfaction and design the hierarchy structure including the main attributes and sub-attributes for supplier selection. The weights of the attributes and alternatives are calculated using pair wise comparison matrices.

In 2012, a combination of fuzzy AHP and fuzzy objective linear programming is used to select the best supplier to develop a low carbon supply chain [29]. At first, Fuzzy AHP is used to determine weights of predetermined criteria which are cost, quality, rejection percentage, late delivery percentage, green house gas emission and demand. Then, by the help of fuzzy objective linear programming, the best supplier is determined.

In 2013, an interactive solution approach is proposed for multiple objective supplier selection problems with Fuzzy AHP [30]. Their methodology includes three objectives; minimizing total monetary cost, maximizing total quality and maximizing service level. By the provided interactivity, the decision maker has the opportunity to incorporate his preferences during the iterations of the optimization process.

Another favorable technique for solving (MCDM) problems is the TOPSIS (technique for the order performance by similarity to ideal solution). TOPSIS, which is a widely accepted multi attribute decision making tool was used [31]. The concept of TOPSIS is that the most preferred alternative should not only have the shortest distance from the positive ideal solution (PIS), but should also be farthest from the negative ideal solution (NIS) [32]. This method is rational and understandable and the computation is uncomplicated. Hwang and Yoon [33] also described the TOPSIS concept, referring to the positive and negative ideal solutions as the ideal and anti-ideal solutions respectively. Numerous applications of TOPSIS exist, including airline performance evaluation [34] and optimal material selection [35].

Based on previous comprehensive literature review, considering multi criteria structure of the supplier evaluation problem and the vagueness in real environment, fuzzy AHP is thought to be a suitable and simple enough for obtaining weights of suppliers. Using TOPSIS as another method for comparing results of fuzzy AHP when solving by fuzzy preference programming (FPP). Mikhailov [36],[37] developed a fuzzy preference programming method, which also derives crisp weights from fuzzy comparison matrices.

2 ANALYTICAL MODEL FOR SUPPLIERS EVALUATION BY FUZZY ANALYTICAL HIERARCHY PROCESS (F-AHP).

Fuzzy Analytical Hierarchy Process (F-AHP) embeds the fuzzy theory to basic Analytic Hierarchy Process (AHP), which was developed by Saaty [8]. The main goal of this analytical model is to develop the best supplier selection and to facilitate the aim of the evaluation model through implementing the Fuzzy Analytical Hierarchy Process, which was a combination of AHP and Fuzzy Theory in order to deal with the uncertainties and vagueness of decision makers' judgments. The approach of fuzzy analytical hierarchy process(F-AHP) have many steps as the following sections.

2.1 Define Criteria for Supplier Selection

The first step in any supplier evaluation procedure is to establish the criteria to be used to evaluate the supplier performance. So every organization selects the most important criteria which meet the strategy of procurement. Choosing many criteria makes supplying process to supply chain more reliable through delivery performance criteria and quality criteria, flexible through service consistency criteria and lowering cost as much the company can through total cost criteria. Therefore, the important criteria have been selected to achieve the goal which is low cost at reasonable response system.

2.1.1 Define Sub Criteria for Supplier Selection

In this step the analysis or decomposing the last selected criteria to the fundamental elements. Total cost criteria will be analyzed to price cost, ordering inventory cost, holding inventory cost and transportation cost. For Quality criteria will be decomposed to sub criteria consisting of defect percentage, technical level requirement and reliability. Delivery performance criteria will be decomposed to sub criteria consisting of lead time and on-time delivery. Service consistency criteria will be decomposed to sub criteria consisting of supply capacity and warranty period .

2.1.2 Structure of Hierarchical Model

The hierarchical model will consist of multi levels. First level acts as the goal of model. Second level consists of last selected criteria for evaluation process which is total cost, quality, delivery performance and service consistency. Third level consists of sub-criteria, while fourth level which has the multi alternatives for evaluation process which consist of multi suppliers, as shown in figure(1) .

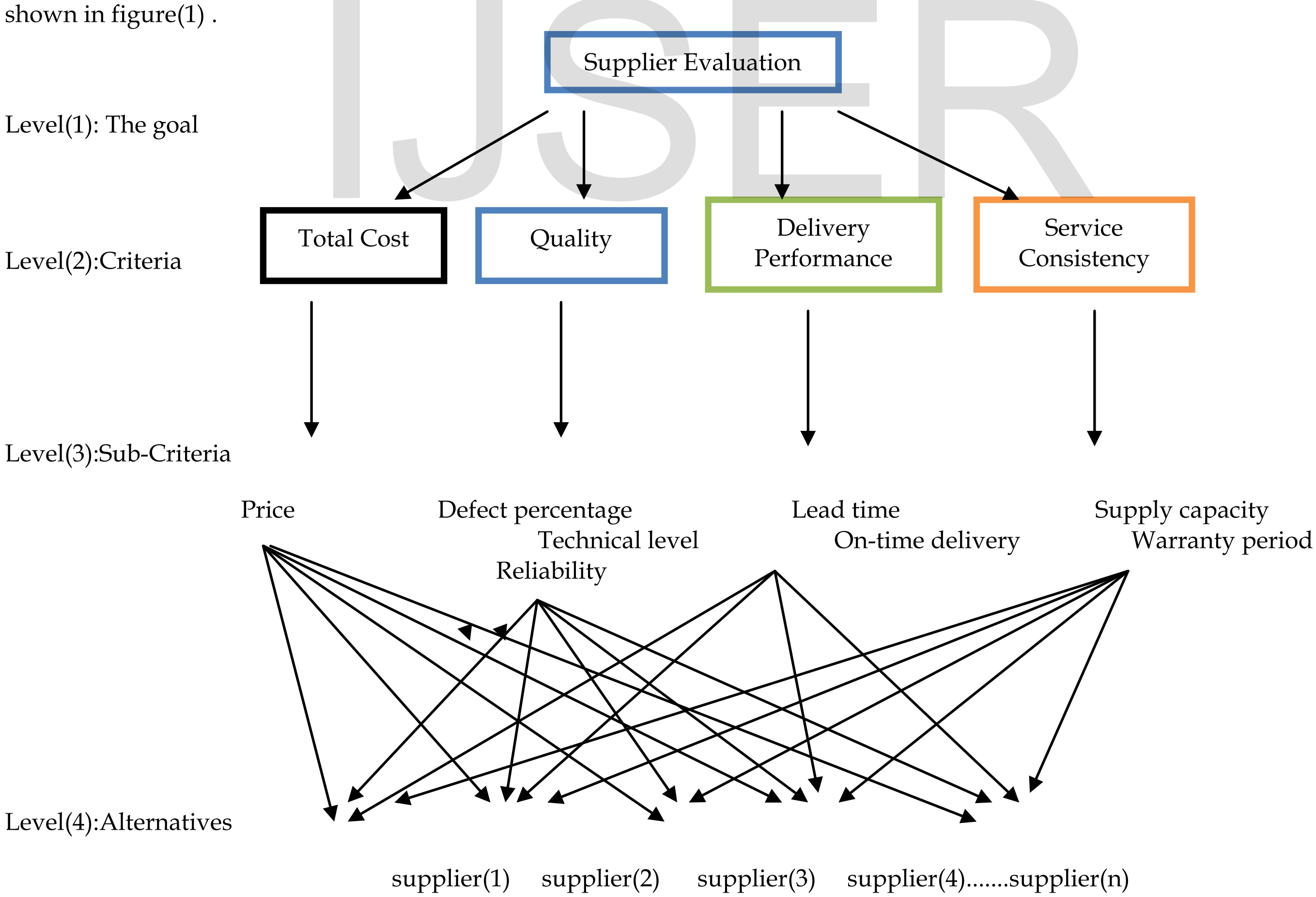

Fig (1). Structure of the hierarchical model

2.1.3 Applying fuzzy theory for pair wise comparison judgments

The Application of fuzzy theory in formulating judgments is very important to tackle the vagueness of qualitative evaluation for any model, since to formulating the judgments into fuzzy linguistic scales covering all expected values (fuzzification the performance and weights values) which may actually occurs.

Let consider :

a_{ij}: the performance of alternative (i) for criteria (j)

w_j: weight or the value of relative importance for criteria (j)

l : smallest possible value
m : the most promising value
u : the largest possible value

(l, m, u) — That describes fuzzy event which may occur.

2.1.3.1 Triangular fuzzy number

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function, which assigns to each object a grade of membership ranging between zero and one. A Triangular Fuzzy Number (TFN) "M" is denoted simply as (l, m, u). The parameters l, m and u, respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event. Each TFN has linear representations on its left and right sides such that its membership function can be defined as in figure (2).

$u_M(x)$: It's a membership function for a triangular fuzzy number .

$$u_M(x) = \begin{cases} (x-l)/(m-l) \quad l \leq x \leq m & \text{if } x \in [l, m] \\ (u-x)/(u-m) \quad m \leq x \leq u & \text{if } x \in [m, u] \\ 0 & \text{otherwise} \end{cases}$$

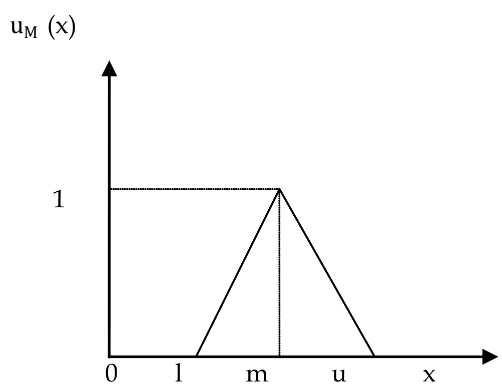

Fig (2). A triangular fuzzy number "M" .

2.1.4 Establishing fuzzy pair wise comparison judgments matrices

The decision making process involves fuzzy linguistic scales for relative importance as given in Table (1) to develop the comparisons between criteria and each other. Involving linguistics describes the degree of importance, for example the comparison of both of criteria cost and delivery performance are compared using the question "How important is the cost when it is compared with the delivery performance at the supplier evaluation decision ?" and the answer for example is "important (I)", so this linguistic scale is placed in the relevant cell against the triangular fuzzy numbers (4,5,6). Therefore the fuzzy pairwise comparison judgments matrices by decision maker are produced in the same manner.

TABLE (1) LINGUISTIC SCALES FOR RELATIVE IMPORTANCE

Linguistic scale for importance	Triangular fuzzy Scale	Triangular fuzzy reciprocal scale
Equally important (EI)	(1,1,1)	(1,1,1)
Slightly important (SM)	(1,2,3)	(1/3,1/2,1)
Fairly important (FI)	(2,3,4)	(1/4,1/3,1/2)
Intermediate (IM)	(3,4,5)	(1/5,1/4,1/3)
Important (I)	(4,5,6)	(1/6,1/5,1/4)
More important (MI)	(5,6,7)	(1/7,1/6,1/5)
Much important (MI)	(6,7,8)	(1/8,1/7,1/6)
Strongly important (SI)	(7,8,9)	(1/9,1/8,1/7)
Absolutely important (AI)	(9,9,9)	(1/9,1/9,1/9)

2.1.5 Solving fuzzy comparison matrix judgments by fuzzy preference programming (FPP).

Calculating the local weights of criteria and sub- criteria pairwise fuzzy comparison matrices by decision maker using the linguistic scales for relative importance by Fuzzy Preferences Programming (FPP) method by Mikhailov [36] , [37] was applied.

2.1.5.1 Fuzzy preference programming method

Mikhailov proposed the FPP method [36],[37] to derive priority vectors from a set of crisp or interval comparisons. The assessment of the priorities is an optimization problem, maximizing the decision-maker's satisfaction with a specific crisp priority vector. Supposing that the decision-maker can provide a set F={ $\tilde{a}_{ij}$} of m ≤ n(n-1)/2 fuzzy comparison judgments, i=1, 2, ..., n-1; j=2, 3,......, n; j > i, represented as triangular fuzzy numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$.

- The problem is to derive a crisp priority vector $w=(w_1, w_2,...., w_n)^T$, such that the priority ratios w_i/w_j are approximately within the scopes of the initial fuzzy judgments, or

$$l_{ij} \le w_i/w_j \le u_{ij} \quad (1)$$

Each crisp priority vector (w) satisfies the double-side inequality (1) with some degree, which can be measured by a membership function, linear with respect to the unknown ratio (w_i/w_j).

$$u_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \left(\left(\frac{w_i}{w_j}\right) - l_{ij}\right)/m_{ij} - l_{ij}, & \left(\frac{w_i}{w_j}\right) \le m_{ij} \\ \left(u_{ij} - \left(\frac{w_i}{w_j}\right)\right)/u_{ij} - m_{ij}, & \left(\frac{w_i}{w_j}\right) \ge m_{ij} \end{cases} \quad (2)$$

The membership function (2) is linearly increasing over the interval $(-\infty, m_{ij})$ and linearly decreasing over the interval (m_{ij}, ∞). The function takes negative values when $\left(\frac{w_i}{w_j}\right) < l_{ij}$ or $\left(\frac{w_i}{w_j}\right) > u_{ij}$, and has a maximum value u_{ij}=1 at $\left(\frac{w_i}{w_j}\right) = m_{ij}$.
Over the range (l_{ij}, u_{ij}), the membership function (2) coincides with the fuzzy triangular judgment (l_{ij}, m_{ij}, u_{ij}).

The solution to the prioritization problem by the FPP method is based on two main assumptions:

1- The first one requires the existence of non-empty fuzzy feasible area P on the (n-1) dimensional simplex Q^{n-1}

$$Q^{n-1} = \left\{(w_1, w_2, \dots\dots, w_n) \middle| w_i > 0, \sum_{i=1}^{n} w_i = 1\right\}, \quad (3)$$

defined as an intersection of the membership functions, similar to (2) and the simplex hyper plane (3). The membership function of the fuzzy feasible area is given by

$$u_p(w) = {}^{\min}_{ij} \{u_{ij}(w) | i = 1,2, \dots, n-1; j = 2,3, \dots, n; j > i\}. \quad (4)$$

If the fuzzy judgments are very inconsistent, then u_p (w) could take negative values for all normalized priority vectors $w \in Q^{n-1}$.

2- The second assumption of the FPP method specifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function (4). It can easily be proved that u_p (w) is a convex set, so there is always a priority vector $w^* \in Q^{n-1}$ that has a maximum degree of membership

$$\lambda^* = u_p(w*) = \max_{w \in Q^{n-1}} \min_{ij} \{u_{ij}(w)\} \quad (5)$$

The maximum prioritization problem (5) can be represented in the following way :

$$\max \lambda$$
$$\lambda \le u_{ij}(w), i = 1,2, \dots., n-1; j = 2,3, \dots., n; j > i.$$
$$\textstyle\sum_{k=1}^{n} w_k = 1, \; w_k > 0, k = 1,2, \dots., n. \quad (6)$$

Taking the specific form of the membership functions (2) into consideration, the problem (6) can be further transformed into a bilinear program.

$$\left.\begin{array}{c} \max \lambda \\ (m_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j \le 0, \\ (u_{ij} - m_{ij})\,\lambda w_j + w_i - u_{ij} w_j \le 0, \\ \sum_{k=1}^{n} w_k = 1, \quad w_k > 0, \; k = 1,2, \dots., n. \\ i = 1,2, \dots., n-1; j = 2,3, \dots., n; j > i. \end{array}\right\} \quad (7)$$

The optimal solution to the non-linear problem above (w*, λ*) might be obtained by employing some appropriate numerical method for non-linear optimization. The optimal value λ* can be used for measuring the consistency of the initial set of fuzzy judgments.

3 Analytical model for suppliers evaluation by Technique for Order Preferences by Similarity to Ideal Solution (TOPSIS).

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) was proposed by Hwang and Yoon [38]. The basic concept of this method is that the selected alternative should have the shortest Euclidean distance from the positive-ideal solution (PIS) and the farthest Euclidean distance from the negative-ideal solution (NIS) in some geometrical sense.

3.1 TOPSIS processes :

The TOPSIS method evaluates the following decision matrix which refers to (m) alternatives which are evaluated in terms of (n) criteria:

(Step 1) Form a decision matrix (D) The structure of the matrix can be expressed as follows:

$$D = \begin{bmatrix} & X_1 & X_2 & . & . & X_j & . & . & X_n \\ A_1 & x_{11} & x_{12} & . & . & x_{1j} & . & . & x_{1n} \\ A_2 & x_{21} & x_{22} & . & . & x_{2j} & . & . & x_{2n} \\ . & . & . & & & . & & & . \\ . & . & . & & & . & & & . \\ A_i & x_{i1} & x_{i2} & . & . & x_{ij} & . & . & x_{in} \\ . & . & . & & & . & & & \\ . & . & . & & & . & & & \\ A_m & x_{m1} & x_{m2} & . & . & x_{mj} & . & . & x_{mn} \end{bmatrix}$$

A_i = i th alternative
X_{ij} = the numerical outcome of the i th alternative with respect to j th criteria

(Step 2) Construct the normalized decision matrix:
An element (r_{ij}) of the normalized decision matrix (R) can be calculated as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}}$$

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{ij} & r_{1n} \\ r_{21} & r_{22} & r_{ij} & r_{2n} \\ \dots & \dots & \dots & \dots \\ r_{m1} & r_{m2} & r_{mj} & r_{mn} \end{bmatrix}$$ Where i=1,2,...,m j=1,2,....,n

(Step 3) Establishing entropy model to compute relative weights of criteria:
The entropy model is used to calculate the elements of evaluation criteria weights. According to the degree of index dispersion, the weight of all indicators (criteria) is calculated by information entropy.
Let the decision matrix (D), number of alternatives (m) and number of criteria (n) and the calculations of entropy measure for every index (criteria) using following equation:

calculate feature weight $p_{ij} = x_{ij} / \sum_{i=1}^{m} x_{ij}$

where $1 \leq i \leq m$

$1 \leq j \leq n$

calculate the output entropy $E_j = -K \sum [p_{ij} \times \ln p_{ij}]$

Where, $K = 1/\ln m$

To generate the divergence vector (d_j), where the divergence can be defined as the degree of diversity of information involved in the outcomes of the j th criterion. $(d_j = 1 - E_j)$
Obtain the normalized weights of indexes (criteria) as $w_j = d_j / \sum_{j=1}^{n} d_j$

(Step 4) Calculate the weighted normalized decision matrix:
Construct the weighted normalized decision matrix, this matrix 'V' can be calculated by multiplying each column of the matrix R with its associated weight w_j. The weighted normalized value v_{ij} is calculated as:

$$V=\begin{bmatrix} w_1 r_{11} & w_2 r_{12} & w_j r_{ij} & w_n r_{1n} \\ w_1 r_{i1} & w_2 r_{i2} & w_j r_{ij} & w_n r_{in} \\ \dots & \dots & \dots & \dots \\ w_1 r_{m1} & w_2 r_{m2} & w_j r_{mj} & w_n r_{mn} \end{bmatrix} \qquad V=\begin{bmatrix} v_{11} & v_{12} & v_{ij} & v_{1n} \\ v_{i1} & v_{i2} & v_{ij} & v_{in} \\ \dots & \dots & \dots & \dots \\ v_{m1} & v_{m2} & v_{mj} & v_{mn} \end{bmatrix}$$

(Step 5) Calculate the positive ideal solution (PIS) and negative ideal solution (NIS):

Positive ideal solution,

$$A^+ = \left(\max._i v_{ij} \middle| j \in J\right), \left(\min._i v_{ij} \middle| j \in J'\right) | i = 1,2, \dots, m$$

$$A^+ = \{v_1^+, v_2^+, \dots \dots, v_j^+, \dots \dots, v_n^+\}$$

Negative-ideal solution,

$$A^- = \left(\min._i v_{ij} \middle| j \in J\right), \left(\max._i v_{ij} \middle| j \in J'\right) | i = 1,2, \dots, m$$

$$A^- = \{v_1^-, v_2^-, \dots \dots, v_j^-, \dots \dots, v_n^-\}$$

Where

J={j =1,2,...,n | j associated with benefit criteria}

and

$J' = \{j = 1,2, \dots, n | j \text{ associated with cost criteria } \}$

(Step 6) Calculate the separation measure. The separation of each alternative for positive-ideal and negative-ideal solution can be measured by Euclidean distance:

$$s^+ = \sqrt{\sum_{j=1}^{n}(v_{ij} - v^+)^2} \qquad \text{where } 1 \le i \le m,\ 1 \le j \le n$$

$$s^- = \sqrt{\sum_{j=1}^{n}(v_{ij} - v^-)^2} \qquad \text{where } 1 \le i \le m,\ 1 \le j \le n$$

where: s^+ and s^- are the separation measure of the i^{th} alternative from A^+ and A^- respectively.

(Step 7) Calculate the relative closeness to the positive ideal solution: Determine the relative closeness of A_i with respect to A^+ (positive ideal solution). This can be measured by the relation:

$$s_i = \frac{s^-}{s^+ + s^-}$$

(Step 8): Rank the preference order, according to the largest value of s_i is the better alternative.

4 MODEL APPLICATION :

The Fuzzy AHP methodology and TOPSIS method are applied in purchasing decision for an industrial company which produces solar heaters. The purchasing department of company adopted purchasing criteria and measures for supplier selection decision. These criteria are (price, quality, delivery performance and service consistency. This is to determine the best supplier among 4 alternatives suppliers producing product (x) needed for this industrial company and regarding 8 criteria as shown in the table (2).

TABLE (2) : THE QUANTITATIVE AND QUALITATIVE INFORMATION FOR SET OF SUPPLIERS PROVIDING PRODUCT (X).

Supplier number	Price($)	Tech. level (grade)	Defects (rate)	Reliability (rate) (%)	On-time delivery (rate)	Supply Capacity (NO. Of parts)	Warranty Period (month)	Lead time (weeks)
1	55	2	0.04	80	0.85	400	4	8
2	40	1	0.01	95	0.95	700	3	12
3	45	1	0.02	90	0.98	600	3	10
4	50	3	0.06	70	0.90	500	4	9

4.1 Application of F-AHP method for solving supplier selection problem

4.1.1 Determining Weights of Criteria

- In order to determine the criteria and evaluate the alternatives for the supplier selection process, a meeting was performed with both production manager and purchasing manager. According to their preferences, the establishing pair wise comparison matrix judgments for criteria decision maker compares the criteria or alternatives via linguistic terms showed in table (1). According to their preferences (the production manager and purchasing manager), the purchasing department decide the price is important than service consistency, quality is important than service consistency and delivery performance and quality are somewhat important than price while delivery performance is somewhat important than service consistency. The averaged pair wise comparison of the criteria is represented by the following tables.

TABLE (3): COMPARISON JUDGMENTS MATRICES FOR MAIN CRITERIA

criteria	Price	Quality	Service Consistency	Delivery performance	Local weights
Price	(1, 1, 1)	(1/3, 1/2, 1)	(2, 3, 4)	(1, 2, 3)	0.2741
Quality	(1, 2, 3)	(1, 1, 1)	(2, 3, 4)	(1 ,2, 3)	0.3744
Service Consistency	(1/4, 1/3, 1/2)	(1/4, 1/3, 1/2)	(1, 1, 1)	(1/3, 1/2, 1)	0.1081
Delivery performance	(1/3, 1/2, 1)	(1/3, 1/2, 1)	(1,2,3)	(1, 1, 1)	0.2435

- The local weights for sub-criteria are shown in the following tables.

TABLE (4): COMPARISON JUDGMENTS MATRICES FOR SUB- CRITERIA OF QUALITY

Quality	Tech. level	Defects	Reliability	Local weights
Tech. level	(1, 1, 1)	(1, 1, 1)	(1/3, 1/2, 1)	0.2500
Defects	(1, 1, 1)	(1, 1, 1)	(1/3, 1/2, 1)	0.2500
Reliability	(1, 2, 3)	(1, 2, 3)	(1, 1, 1)	0.5000

TABLE (5): COMPARISON JUDGMENTS MATRICES FOR SUB- CRITERIA OF DELIVERY PERFORMANCE

Delivery performance	Lead time	On-time delivery	Local weights
Lead time	(1, 1, 1)	(2, 3, 4)	0.7500
On-time delivery	(1/4, 1/3, 1/2)	(1, 1, 1)	0.2500

TABLE (6): COMPARISON JUDGMENTS MATRICES FOR SUB- CRITERIA OF SERVICE CONSISTENCY

Service Consistency	Supply capacity	Warranty period	Local weights
Supply capacity	(1, 1, 1)	(1/4, 1/3, 1/2)	0.7500
Warranty period	(2, 3, 4)	(1, 1, 1)	0.2500

4.1.2 Determining Weights of Alternative Suppliers with Respect to Criteria

After achieving the normalized non-fuzzy relative weights for criteria, the same methodology is applied to find the respective values for alternative suppliers. But, the alternatives should be pair wise compared with respect to each criterion particularly as shown in table (7). Quantitative and qualitative information about each criterion and sub-criterion for each supplier are needed to establish judgments pair wise comparison.

TABLE (7): COMPARISON JUDGMENTS MATRICES FOR PERFORMANCE OF SUPPLIER RESPECT TO CRITERIA AND SUB-CRITERIA .

Price (C1)	S1	S2	S3	S4	Local weight
S1	(1,1,1)	(1/8,1/7,1/6)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	0.0667
S2	(6,7,8)	(1,1,1)	(2,3,4)	(4,5,6)	0.5333
S3	(4,5,6)	(1/4,1/3,1/2)	(1,1,1)	(2,3,4)	0.2667
S4	(2,3,4)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	(1,1,1)	0.1333
Tech. level	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(1/4,1/3,1/2)	(1/4,1/3,1/2)	(2,3,4)	0.1653
S2	(2,3,4)	(1,1,1)	(1,1,1)	(4,5,6)	0.3831
S3	(2,3,4)	(1,1,1)	(1,1,1)	(4,5,6)	0.3831
S4	(1/4,1/3,1/2)	(1/6,1/5,1/4)	(1/6,1/5,1/4)	(1,1,1)	0.0685
Defects	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	(2,3,4)	0.1333
S2	(4,5,6)	(1,1,1)	(2,3,4)	(6,7,8)	0.5333
S3	(2,3,4)	(1/4,1/3,1/2)	(1,1,1)	(4,5,6)	0.2667
S4	(1/4,1/3,1/2)	(1/8,1/7,1/6)	(1/6,1/5,1/4)	(1,1,1)	0.0667

Reliability	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	(2,3,4)	0.1333
S2	(4,5,6)	(1,1,1)	(2,3,4)	(6,7,8)	0.5333
S3	(2,3,4)	(1/4,1/3,1/2)	(1,1,1)	(4,5,6)	0.2667
S4	(1/4,1/3,1/2)	(1/8,1/7,1/6)	(1/6,1/5,1/4)	(1,1,1)	0.0667
Lead time	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(4,5,6)	(2,3,4)	(1,2,3)	0.4430
S2	(1/6,1/5,1/4)	(1,1,1)	(1/4,1/3,1/2)	(1/5,1/4,1/3)	0.0793
S3	(1/4,1/3,1/2)	(2,3,4)	(1,1,1)	(1/3,1/2,1)	0.1837
S4	(1/3,1/2,1)	(3,4,5)	(1,2,3)	(1,1,1)	0.2940
On-time delivery	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(1/5,1/4,1/3)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	0.0678
S2	(3,4,5)	(1,1,1)	(1/3,1/2,1)	(2,3,4)	0.3705
S3	(4,5,6)	(1,2,3)	(1,1,1)	(2,3,4)	0.4423
S4	(2,3,4)	(1,1,1)	(1/4,1/3,1/2)	(1,1,1)	0.1195
Warranty period	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(2,3,4)	(2,3,4)	(1,1,1)	0.3750
S2	(1/4,1/3,1/2)	(1,1,1)	(1,1,1)	(1/4,1/3,1/2)	0.1250
S3	(1/4,1/3,1/2)	(1,1,1)	(1,1,1)	(1/4,1/3,1/2)	0.1250
S4	(1,1,1)	(2,3,4)	(2,3,4)	(1,1,1)	0.3750
Supply capacity	S1	S2	S3	S4	Local weights
S1	(1,1,1)	(1/6,1/5,1/4)	(1/4,1/3,1/2)	(1/3,1/2,1)	0.1038
S2	(4,5,6)	(1,1,1)	(1,2,3)	(2,3,4)	0.4154
S3	(2,3,4)	(1/3,1/2,1)	(1,1,1)	(1,2,3)	0.3765
S4	(1,2,3)	(1/4,1/3,1/2)	(1/3,1/2,1)	(1,1,1)	0.1042

4.1.3. Compute the overall score of each supplier

The global priority weights are determined for all criteria and sub-criteria as shown in the "Global weights" column of table (8). The last column of table (8) is the priority order of criteria. It can be seen that price criterion occupies the top-most ranking in the table.

TABLE : (8) COMPARISON PRIORITY WEIGHTS FOR CRITERIA AND SUB-CRITERIA

Criteria	Local weights	Sub-criteria	Local weights	Global weights	Priority order of criteria

Price	0.2741				
Quality	0.3744	Defect percentage	0.2500	0.0936	4
		Tech. level	0.2500		
		Reliability	0.5000		
Delivery performance	0.2435	Lead time	0.7500	0.0608	6
		On-time delivery	0.2500		
Service consistency	0.1081	Supply capacity	0.7500	0.0270	7
		Warranty period	0.2500		

TABLE (9) : FINAL WEIGHTS FOR SUPPLIER EVALUATION PROCESS

Supplier number	Price($)	Defects (rate)	Tech. level (grade)	Reliability (rate) (%)	Lead time (weeks)	On-time delivery (rate)	Supply Capacity (parts)	Warranty Period (month)	Supplier weights
S1	0.0667	0.1333	0.1653	0.1333	0.4430	0.0678	0.1038	0.3750	0.1747
S2	0.5333	0.5333	0.3831	0.5333	0.0793	0.3705	0.4154	0.1250	0.4058
S3	0.2667	0.2667	0.3831	0.2667	0.1837	0.4423	0.3765	0.1250	0.2782
S4	0.1333	0.0667	0.0685	0.0667	0.2940	0.1195	0.1042	0.3750	0.1413

5

APPLICATION OF TOPSIS METHOD FOR SOLVING SUPPLIER SELECTION PROBLEM :

Step 1: Form a decision table.

TABLE (10) : THE STRUCTURE OF THE DECISION TABLE CAN BE EXPRESSED AS FOLLOWS

Supplier number	Price ($) C_1	Tech. level (grade) C_2	Defect (rate) C_3	Reliability (rate) (%) C_4	On-time delivery (rate) C_5	Supply Capacity (parts) C_6	Warranty Period (month) C_7	Lead time (weeks) C_8
S1	55	2	0.04	80	0.85	400	4	8
S2	40	1	0.01	95	0.95	700	3	12
S3	45	1	0.02	90	0.98	600	3	10
S4	50	3	0.06	70	0.90	500	4	9
ΣX_{ij}	190	7	0.1300	335	3.6800	2200	14	39
$\sqrt{\Sigma X_{ij}^2}$	95.66	3.873	0.0755	168.56	1.842	1122.497	7.07106	19.7230

Step 2: Construct the normalized decision table

TABLE (11) : CONSTRUCTION OF THE NORMALIZED DECISION TABLE

Supplier number	Price($) C_1^-	Tech. level (grade) C_2^+	Defects (rate) C_3^-	Reliability (rate) (%) C_4^+	On-time delivery (rate) C_5^+	Supply Capacity (parts) C_6^+	Warranty Period (month) C_7^+	Lead time (weeks) C_8^-
S1	0.5750	0.5164	0.5298	0.4764	0.4615	0.3563	0.5657	0.4056

S2	0.4181	0.2582	0.1325	0.5636	0.5157	0.6236	0.4243	0.6084
S3	0.4704	0.2582	0.2649	0.5339	0.5320	0.5345	0.4243	0.5070
S4	0.5227	0.7746	0.7947	0.4153	0.4886	0.4454	0.5657	0.4563

Step 3: Establishing entropy model to computing relative weights of criteria

TABLE (12) : ENTROPY MODEL RESULTS FOR FEATURE WEIGHTS

Supplier number	Price($) $\mathbf{C_1^-}$	Tech. level (grade) $\mathbf{C_2^+}$	Defects (rate) $\mathbf{C_3^-}$	Reliability (rate) (%) $\mathbf{C_4^+}$	On-time delivery (rate) $\mathbf{C_5^+}$	Supply Capacity (parts) $\mathbf{C_6^+}$	Warranty Period (month) $\mathbf{C_7^+}$	Lead time (weeks) $\mathbf{C_8^-}$
S1	0.2895	0.2857	0.3077	0.2388	0.2310	0.1818	0.2857	0.2051
S2	0.2105	0.1429	0.0769	0.2836	0.2582	0.3182	0.2143	0.3077
S3	0.2368	0.1429	0.1538	0.2687	0.2663	0.2727	0.2143	0.2564
S4	0.2632	0.4286	0.4615	0.2090	0.2446	0.2273	0.2857	0.2308

TABLE (13): FINAL RESULTS FOR ENTROPY MODEL INCLUDING THE NORMALIZED WEIGHTS OF CRITERIA

Index Number (criteria NO.)	The output entropy (E_j)	The degree of diversity (d_j)	The normalized weights of criteria (w_j)
C1	0.9945	0.0055	0.0216
C2	0.9208	0.0792	0.3105
C3	0.8686	0.1314	0.5151
C4	0.9947	0.0053	0.0208
C5	0.9985	0.0015	0.0059
C6	0.9844	0.0156	0.0612
C7	0.9921	0.0079	0.0310
C8	0.9913	0.0087	0.0341

(Step 4) Calculating the weighted normalized decision matrix:

$$v_{ij} = w_j \times R_{ij}$$

$$V_{ij} = \begin{bmatrix} 0.0124 & 0.1603 & 0.2729 & 0.0099 & 0.0027 & 0.0218 & 0.0175 & 0.0138 \\ 0.0090 & 0.0802 & 0.0683 & 0.0117 & 0.0030 & 0.0382 & 0.0132 & 0.0207 \\ 0.0102 & 0.0802 & 0.1364 & 0.0111 & 0.0031 & 0.0327 & 0.0132 & 0.0173 \\ 0.0113 & 0.2405 & 0.4093 & 0.0086 & 0.0029 & 0.0273 & 0.0175 & 0.0156 \end{bmatrix}$$

(Step 5) Calculating the positive ideal solution (PIS) and negative ideal solution (NIS):
The positive- ideal solution A^+ = [0.0090 , 0.2405 , 0.0683 , 0.0117 , 0.0031 , 0.0382 , 0.0175 , 0.0138]
The negative-ideal solution A^-= [0.0124 , 0.0802 , 0.4093 , 0.0086 , 0.0027 , 0.0218 , 0.0132 , 0.0207]

(Step 6) Calculating the separation measure. The separation of each alternative for positive-ideal and negative-ideal solutions can be measured by Euclidean distance:

$$s^{+i} = \sqrt{\sum_{j=1}^{n} (v_{ij} - A^+)^2} \quad \text{where } 1 \le i \le m$$

$$1 \le j \le n$$

$$s^{-i} = \sqrt{\sum_{j=1}^{n}(v_{ij} - A^{-})^2} \quad \text{where} \quad 1 \le i \le m$$

$$1 \le j \le n$$

where: s^{+i} and s^{-i} are the separation measure of the i^{th} alternative from V^+ and V^- respectively.

$s^{+}1 = 0.22040$ $s^{+}2 = 0.16050$ $s^{+}3 = 0.17174$ $s^{+}4 = 0.34120$

$s^{-}1 = 0.08052$ $s^{-}2 = 0.20530$ $s^{-}3 = 0.13701$ $s^{-}4 = 0.16053$

(Step 7) Calculating the relative closeness to the positive ideal solution: Determine the relative closeness of A_i with respect to A^+ (positive ideal solution). This can be measured by the relation:

$$s_i = \frac{s^-}{s^+ + s^-}$$

(Step 8) Ranking the preference order: The largest value of (s_i) is the better alternative.

TABLE (14) FINAL RANKING (SELECTION) OF THE SUPPLIER(S).

Rank	Suppliers	The relative closeness to the positive ideal solution(s_i)
1	S2	0.5612
2	S3	0.4437
3	S4	0.3199
4	S1	0.2675

6 VIEWING THE COMPARISON FOR THE GENERATED RESULTS :

The Fuzzy AHP methodology and TOPSIS method are applied for the same problem as previously illustrated. The results which are generated from both methods are coincide as shown in the table (22).

TABLE (15) THE COMPARISON OF FINAL RANKING (SELECTION) OF THE SUPPLIER(S) GENERATED FROM F-AHP AND TOPSIS.

Suppliers for evaluation process	Results of F-AHP		Results of TOPSIS	
	$*$ (w_i)	Rank	** (s_i)	Rank
S1	0.4058	S2	0.5612	S2
S2	0.2782	S3	0.4437	S3
S3	0.1747	S4	0.3199	S4
S4	0.1413	S1	0.2675	S1

∗Final weights for supplier generated by F-AHP

** The relative closeness to the positive ideal solution

7 CONCLUSION

Supplier selection is a complex multi-criteria decision-making problem. Its complexity is further aggravated if the decision makers preferences depend on both tangible and intangible criteria taken into consideration. The development of a Fuzzy AHP multi-criteria decision making model for suppliers evaluation and selection showed an advantage over other models like the AHP. Adoption of fuzzy numbers, effectively improves the flexibility of the conventional AHP in dealing with the uncertainty and ambiguity associated with different decision makers' judgments. F-AHP proved that it is a proper approach aiming to enhance both efficiency and accuracy of decision maker for handling vagueness problems in evaluation issues such as multi-supplier evaluation. TOPSIS method used for supplier evaluation and the generated ranking for suppliers matched with the same results generated by F-AHP .

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